4.3 Logarithms

- A logarithm base b of a positive number x satisfies the following definition.

  For $x > 0$, $b > 0$, $b \neq 1$, $y = \log_b(x)$ is equivalent to $b^y = x$. where,

- We read $\log_b(x)$ as "the logarithm with base b of x or the log base b of x."

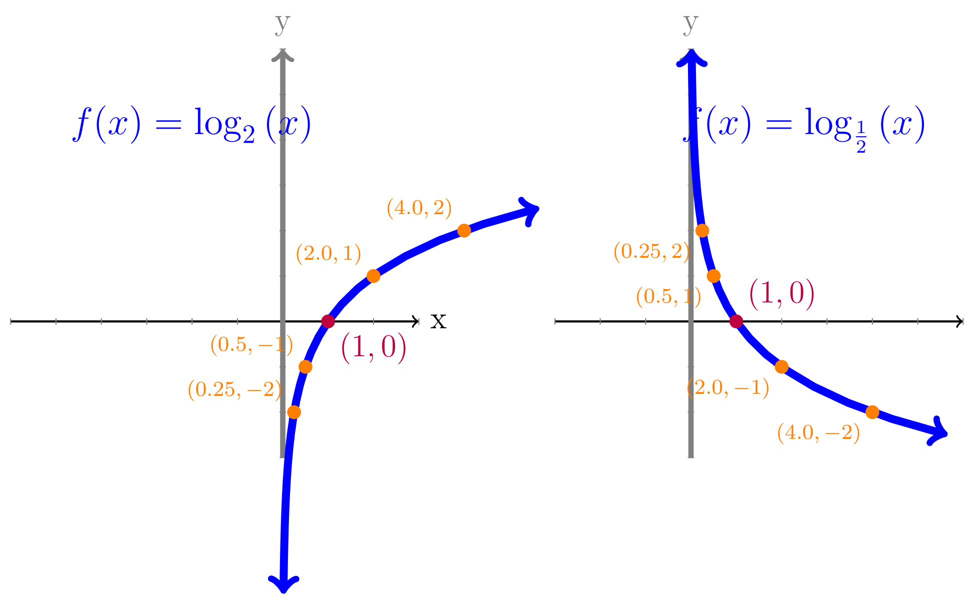
- The logarithm $y = \log_b(x)$ is the exponent to which b must be raised to get x.

- The logarithmic function $y = \log_b(x)$ and exponential function $y = b^x$ are inverses of each other. Since the logarithmic and exponential functions switch the x and y values, the domain and range of the exponential function are interchanged for the logarithmic function. Therefore, the domain of the logarithm function with base b is $(0, \infty)$. the range of the logarithm function with base $b > 0$ is $(-\infty, \infty)$.

$$\log_b(x) = y$$

$$b^y = x$$

- **A few graphs:**



- The difference between graph of log functions in base $b > 1$ and $0 < b < 1$ is illustrated in the above graphs.

- The logarithm with natural base e is denoted by ln. That is, $\ln(x) = \log_e(x)$.

- The logarithm base 10 is denoted by log, omitting the base. That is, $\log(x) = \log_{10}(x)$

1. Evaluate.

(a) $\log_3(3) =$

(b) $\log_3(81) =$

(c) $\log_9(81) =$

(d) $\log_9(3) =$

(e) $\log_9\left(\frac{1}{3}\right) =$

(f) $\log_4(8) =$

(g) $\log_2(1024) =$

(h) $\log_2(.5) =$

(i) $\log_4(\sqrt{2}) =$

(j) $\log_2(\sqrt{2}) =$

(k) $\log(10,000) =$

(l) $\log(0.1) =$

2. Solve for x.

(a) $\log_3(x) = 2$

(b) $\log(x) = 5$

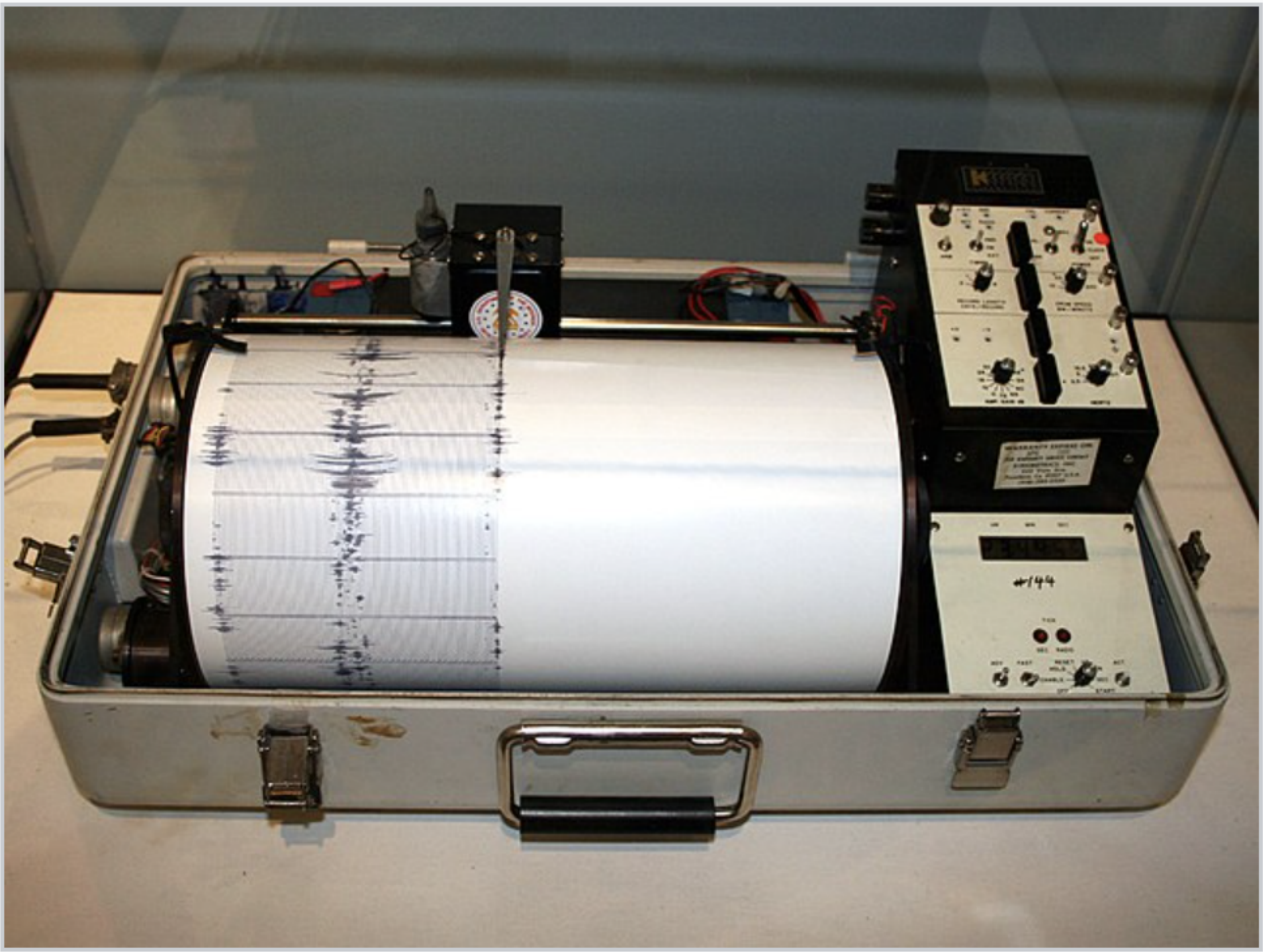
(c) $x = \ln(e^2)$

(d) $x = \ln(\sqrt{e})$

3. **Mechanical and Civil Engineering:** The intensity levels I of two earthquakes measured on a seismograph can be compared by the formula $\log\left(\frac{I_1}{I_2}\right) = M_1 - M_2$ where M_1 and M_2 are the magnitudes given by the Richter Scale.

(a) How many times more intense is an earthquake of 7.1 than an earthquake of 5.5 Richter?

(b) If an earthquake is 70 times as intense as another, what is the difference of their Richter scale magnitude?



Kinemetric seismograph.

Figure 1: Wikipedia